UNIVERSITY OF DELHI

SCHEME OF EXAMINATION
AND
COURSES OF READING
FOR
B.A./B.Sc. (HONOURS) EXAMINATION IN MATHEMATICS

Part I Examination 1989

Part II Examination 1990

Part III Examination 1991

Syllabi applicable for students seeking admission to the B.A./B.Sc. (Hons.) Mathematics Course in the Academic year 1989-90

Price : Rs. 2 -

UNIVERSITY OF DELHI

B.A. & B.Sc. (Hons.) Mathematics

SCHEME OF EXAMINATION

The Courses are divided into three parts. Part I is to be covered in the first year, Part II in the second year and Part III in the third year. The contents of the courses have been divided into ten papers. Details of the syllabi for each paper are given in the following pages.

	Duration (Hrs.)	Marks
Part I Examination—1989		
Paper I—Algebra and Analytic Geometry	3	100
Paper II—Calculus	3	100
Part II Examination—1990		
Paper III—Analysis I	3	100
Paper IV—Algebra I	3	100
Paper V—Differential Equations and Mechanics I	3	100
Part III Examinations—1991		
Paper VI—Analysis II	3	100
Paper VII—Algebra II	3	100
Paper VIII—Differential Equations and Mechanics II	3	100

Paper IX & X—Any two of the following : 3+3 100+100

(i) Number Theory

(ii) Mathematical Statistics

(iii) Numerical Mathematics

(iv) Linear Programming and Theory of Games

(v) Lattice Theory

(vi) Probability Theory

(vii) Computer Mathematics

Note :—Those who offer Mathematical Statistics as a subsidiary subject *will not be* allowed to offer Mathematical Statistics as an optional course.

Part I—Examination

Paper I—Algebra and Analytic Geometry

Algebra (2/5)

System of complex numbers introduced as a system of ordered pairs of real numbers Representation of the line segment, straight line, circle and regions in the complex plane. De Moivre's Theorem Applications to the determination of roots of complex numbers, expressions for $\sin n\theta$ and $\cos n\theta$ in terms of $\sin\theta$ and $\cos\theta$ & vice versa. Sums of simple finite series like $\Sigma \cos n\theta$, $\Sigma \sin n\theta$ etc.

Algebra of Matrices. Rank of a matrix and its invariance under elementary row and column transformations. Solutions of systems of linear equations with not more than four unknowns.

Symmetric and Skew symmetric matrices. Hermitian and Skew-Hermitian matrices

Relations between roots and coefficients of a polynomial equation. Evaluation of symmetric functions of roots of cubic and biquadratic equations.

Analytic Geometry (3/5)

Second and higher degree equations representing straight lines. Bisectors of a pair of intersecting lines.

Coaxial circles. Pole and Polar and related properties. Orthogonal circles, Equations of parabola, ellipse and hyperbola in standard forms. Elementry properties of these curves. Change of coordinate axes. Classification of Curves represented by an equation of the second degree in two variables.

Analytic study of plane, staight lines, sphere, cone and cylinder. Standard equations of ellipsoid and elementary properties.

Paper II—Calculus 100 Marks

[*Revision Unit* : The real number, system, Representation of real numbers as points on the real line, the notion of distance and an interval on the real line. The concept of real functions and their graphs. Derivatives of polynomial and trigonometric functions and of their simple combinations. Primitives of functions and their calculation in simple cases. Integration by substitution and by parts. Applications to determination of areas under place curves in simple cases].

Neighbourhood of a point on the real line. Notion of limit of a function, Algebra of Limits Continuity of a function at a point and on an interval, example of continuous and discontinuous functions with geometrical illustrations, algebra of continuous functions, composition of continuous functions.

Derivatives of a function at a point and on an interval, geometrical interpretation of derivative, derivative as a rate measure, algebra of derivable functions, composite functions of derivable functions. Inverse functions and their derivatives, derivative of Implicit functions, and derivatives of function defined parametrically. Derivatives of logarithmic, exponential, and inverse trigonometric function, Hyperbolic and Inverse hyperbolic functions and their derivatives. Derivatives of higher orders and Leibnitz rule.

Tangents and normals, sub-tangents and subnormals, curvature, radius of curvature, circle of curvature, involutes and evolutes and asymptotes of curves in cartesian and polar co-ordinates. Singular points and curve tracing. Functions on R^2 to R, partial differentiation, Euler's theorem on homogenous functions.

Integration by partial fractions, integration of rational and irrational functions, Properties of definite integrals. Reduction formulae.

Evaluation of areas and lengths of curves in the plane, evaluation of volumes and surfaces of solids of revolution.

Part II Examination

Paper III—Analysis I 100 Marks

Sequences of real numbers, convergent sequences, Cauchy sequences, algebras of convergent and Cauchy sequence. Cantor's construction of real numbers (starting from rational numbers). Cauchy's general principle of convergence. Bounded sets of real numbers, suprema, infima, existence of suprema and infima of bounded sets. Monotonic sequences, limit superior, limit inferior of sequences. Infinite series and their convergence. Comparison test, root test, ratio test, Raabe's test, integral test, Leibnitz test and Dirichlet's test for convergence of series. Absolute convergence and rearrangement of series. Convergence and absolute convergence of double series, sufficient conditions for the validity of

$$\sum_{m\,n} a_{m,n} = \sum_{m}\sum_{n} a_{m,n} = \sum_{n}\sum_{m} a_{m,n}.$$

Products of two absolutely convergent series. Cauchy product of two series, one of which is absolutely convergent. Real numbers and Decimal representations.

Properties of continuous functions uniform continuity, discontinuous functions, types of discontinuity and discontinuty of monotonic functions, Infinite Limits and Limits at infinity.

Rolle's theorem, mean value theorem, Taylor's theorem with Lagrange's and Cauchy's form of remainder, Taylor's and Maclaurin's series of elementary functions. Indeterminate forms.

Functions of two and three variables, their continuity and differentiability. Young's and Schwarz' condition of equality of f_x and f_{yx}. Implicit function theorem. Taylor's theorem and maxima and minima for functions of two variables, Lagrange's method of undetermined muptipliers.

Paper IV—Algebra I 100 Marks

Group Theory : Semigroups, groups, different characterizations of groups. Subgroups, Lagrange's Theorem. Cyclic groups. Normal subgroups. Quotient groups. Homomorphism and Isomorphism theorems, Permutation groups and Cayley's Theorem. Even and odd permutations and A_n.

Rings & Fields : Rings, subrings ideals, Quotient Rings, Integral domains. Division Rings Field. Subfields. Characteristic of a field. Homomorphism and Isomorphism Theorems. Imbedding of a ring without unity in a ring with unity.

Linear Algebra : Vector Spaces, subspaces. Bases and dimension, linear transformations. Algebra of linear transformation. Matrices and linear transformation. Rank and nullity of a linear transformation.

Books Suggested for Reference :

1. Topic in Algebra by I.N. Herstein. Blaisdell Publishing Co—3rd Edition, Vikas Publication 1971.
2. Linear Algebra by K. Hoffman and R. Kunze, Prentice Hall Inc, 1961.
3. Linear Algera by S. Lang Addison—Wesely Publishing Co. 1968.
4. A First Courses in Abstract Algebra by J.B. Fraleigh. Addison, Wesley Publishing Co.—1968.

Paper V—Differential Equations and Mechanics I 100 Marks

Mechanics :

(Review Unit : Scalar and vector products ; projection of a vector on a directed line).

Triple products. Differentiation and integration of a vector function on an interval Differentiation of a product of two vectors, Gradient, divergence and curl of a vector. Moments of a (localised) vector, about a point Scalar moment of a vector about a directed line.

Basic concepts of mechanics. Basic laws of mechanics Interial frames of reference. Work and Energy. Principles of linear momentum, angular momentum and, energy for a particle Conservation field and polential energy. Principle of conservation of energy for a particle.

Rectilinear motion : Uniformly accelerated motion (including connected system). Resisted motion. Harmonic Oscilator. Damped and forced vibrations. Elastic springs and strings.

Hooke's law. Vertical and horizontal vibrations of a paticle attached to an elastic string.

Motion in a plane : Components of velocity and acceleration : Cartesian, radical and transverse; tangentil and normal. Projectile motion in a non-resisting medium. Constrained motion in a horizontal circle conical pendulum Constrained motion on a smooth vertical circle. Simple pendulum, Motion of a particle under a central force. Differential equcation of a central orbit in both reciprocal polar and pedal coordinates, Newton's law of gravitation and planetary orbits. Kepler's laws of motion deduced from Newton's law of gravitation and vice versa.

Coplaner force systems. Necessary and sufficient condition for equilibrium of a particle. Triangle law of forces, polygon law of forces and Lami's theorem.

Moment of a force about a line. Varignen's theorem for concurrent force systems. Necessary condition for a system of particles to be in equilibrium.

Equipollent force system—definition. Couples, moment of couple, equipollence of two couples. Reduction of a general plane force system. Parallel force systems. Centre of gravity formulae. use of symmetry and standard results (Statements only). Principle of virtual work of a system of particles.

Motion of a system of particles in plane. Motion of the mass centre and motion relative to the mass centre. Principles of linear momentum, angular momentum and energy for a system. Two body problem.

Infinitesimal displacement of a plane lamina. Necessary and sufficient conditions for equilibrium rigid body, movable Parallel to a fixed plane. Problems on equilibrium under forces including friction (excluding indeterminate cases). Stable equilibrium, Energy test of stability (problems involving one variable only).

Differential Equations :

First order differential equations. Second order differential educations with constant and variable coefficients.

Homogeneous linear differential equations. Systems of linear differential equations.

Books Suggested for Reference :

1. Principles of Mechanics by Synge and Griffiths.
2. A text book of Dynamics by F. Chorlton—Chapters 3-6 for problems.
3. Statics by A.S. Ramsey—Chapters 3-6, 9, 11-12 for problems.

Part III Examination

Paper VI—Analysis II

Definition and existence of Riemann integral of a bounded function, Darboux condition of integrability. Riemann integrability of continuous functions and monotonic functions. Riemann integral of functions with finite number of discontinuities and of functions with discontinuity points having a finite number of limit points. Reimann integral as the limit of a sum. The Fundamental theorem of Integral calculus, Mean value theorems. Definition and examples of Riemann-Stieltjes integral of bounded functions.

Sequences and series of functions and their pointwise convergence. Uniform convergence of sequences and series of functions, Weierstrass M-test Uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, Weierstrass approximation theorem.

Power series and their convergence. Absolute and uniform convergence of a power series. Definitions of exponential, logarithmic, trigonometric functions by means of power series and deduction of their properties.

Fourier series and its convergence. Fourier series of functions of bounded variation and differentiable functions.

Improper integrals. Convergence of an Improper integral Comparison tests. Dirichlet's test. Beta and Gamma functions, their properties and relationships. Differentiation under Integral sign.

Double and triple integrals, iterated integrals, change of order of Integration, Line, Surface and Volume Integrals. Green's, Gauss' and Stoke's Theorem.

Paper VII—Algebra II

100 marks

Group Theory : Centre, Normalizer, conjugacy Class Equation. Finite groups. Cauchy's and Sylow's Theorems. Automorphisms. Inner Automorphisms. Direct product of two groups.

Rings : Imbedding of an interal domain in a field. Field of Quotients. Polynomials over commutative rings. Prime and maximal ideals in commutative rings. Eculidean domains. Principal ideal domains, Unique factorisotion domains Eisenstein's criterion of irreducibility.

Linear Algebra : Dual spaces. transpose of a linear transformation. Direct sum of subspaces. Characteristic values, Characteristic vactors, Cayley-Hamilton Theorem. Inner product space. Gram-Schmidt orthogonalization process.

Books suggested for Reference :

1. Topics in Algebra by I.N. Herstein. Blaisdell Publishing, Co—3rd Edition.
2. Linear Algebra by K. Hoffman and R. Kunze; Prentice Hall Inc. 1961.
3. Linear Algebra by S. Lang, Addison-Wesley Publishing Co. 1968.
4. A First courses in Abstract Algebra by J.B. Fraleigh Addison Wesley Publishing Co—1968.

Marks 100

Paper VIII—Differential Equations and Mechanics II

Mechanics :

General force system—total force and total moment relative to a base point Total moment under a change of base point. Necessary and sufficient conditions for a system to be equipollent to zero. Moment of a couple, composition of couples. Reduction of a force system to a force and a couple. Reduction to a wrench. Invariants of a system.

Euler's theorem (without proof) on displacement of rigid body with one point fixed, General displacement of a rigid body. Infinitesimal displacement of a rigid body about a point. Composition of infinitesimal displacements. Reduction to a screw displacement.

Work done on (i) a particle (ii) a rigid body, in a given infinitesimal. displacement. Necessary and sufficient conditions for equilibrium of a rigid body (or a system with workless constrains) by an application for the principle of virtual work.

Motion of a boby about a fixed point. Angular velocity. Relation between angular velocity and linear velocity of a point of the body. General motion of a body

Accelerating frams, Rotating frames. Time of a vector referred to rotating frames. Coriolis force and centrifugal force Frames with constant angular velocity.

Moment of inertia : Definitions and stanard results. Momental ellipsoid. Parallel axes and perpendicular axes theorems. Principal axes of inertia, Existence of principal axes of inertia. at a point. Determination of principal axes of inertia Equimomental systems.

Angular momentum and kinetic energy of a rigid body rotating about a fixed point. Kinetic energy of a rigid body in a general motion.

Principles of linear momentum angular momentum and energy for a rigid body. 'D' Alembert's principal and general equations of motion of rigid body. Motion about a fixed axis. Compound pendulum.

Impulse, impulsive forces. Impulsive motion in a plane, elastic impact (direct oblique). Two dimesional problems in rigid body dynamics under finite and impulsive force.

Pressure at a point, Resultant pressure on a plane surface.

Differential Equations :

Total differential equations in three variables.

General, singular and complete solutions of partial differential equations of the first order. Lagrange's method, Charpits's method and Monge's method for partial differential equations of the second order. Linear partial differential equations with constant coefficients. Homogeneous linear partial differential equation with varible coefficients.

Books suggested for Reference :

1. Principles of Mechanics by Synge and Griffith.
2. A text book of Dynamics by F. Chorlton-Chapters 7-8 for problems.
3. Statics by A.S. Ramsey-Chapter 14 for problems.
4. Hydrostatics by A.S. Ramsey.

Paper IX and X—Any two of the following :

Optional (i) Number Theory

The Basic Representation theorem, Linear Diophantine equations, Fundamental theorem of Arithmetic, Fermat's little theorem and Wilson's theorem.

Basic properties of congruences, Residue system, Euler's theorem ; Chinese Remainder Theorem. Multiplicative arithmatic functions, the function ϕ (n), μ (n) ∂ (n) and their simple properties: Mobius Inversion formula. Primitive roots modulon.

Elementary properties of π (x), Legendre's formula for the highest power of a prime number that divides nl, statement of the prime number theorem, Euler's creterion for quadratic residue, the Legendre symbol, the Quardratic Reciprocity law and its applications.

Sums of two and four squares, Fermat's conjecture. Graphical representation of partitions. Euler's partition. theorem.

Books for reference :

G.E. Andrews : 1971, Number Theory.

Option (ii) Mathematical Statistics

Probability : Classical Relative. and Axiomatic Conditional probability and independence.

Random variables, Distribution functions. Mathematical expectation, generating functions and characteristic function.

Discrete Distribution : Binomial, Poisson, Geometric, Negative Binomial, Hypergeometric and Multinomial.

Continuous Distribution : Uniforms, Normal, Exponential, Gamma, Beta, Cauchy, Laplace and their interrelations.

Joint and Conditional distributions, Conditional expectations. Correlation and linear regression for two variables. Joint moment generating function and moments Bivariate normal distribution.

Cumulative-Distribution-Function, Moment-generating-function and Transformation Techniques for finding the distributions of functions of random variables. Expectations of functions of random variables.

Sampling distributions of mean and variance of a random simple from a normal population. Chi-square, F and Student's to distributions.

Weak law of large numbers. Central limit theorem for independent and indentically distributed random variates.

Books Suggested:

1. Introduction to the theory of Statistics by A.M. Mood, F.A. Graybill and D.C. Boes. McGraw-Hill Kogakusha Ltd.. 3rd Edition, 1976 (Chapters I to VI).
2. An introduction to Probability Theory and Mathematical Statistics by V.K. Rohtagi, Wiley Eastern Ltd., 1985.
3. Introduction to Mathematical Statistics by R.V. Hogg and A.T. Craig, McMillan 4th Edition, 1978.

Option, (iii)—Numerical Mathematics :

1. *Interpolation & Approximation:*

Difference operators and their relationship. Lagranges and Newton interpolations. Hermite interpolation. Inverse interpolation. Least square approximation. Kinds of Errors and their control.

2. *Solution of algebraic and transcendental equation*:

Bisection method. Secant, Newton-Rephson, Muller's and Chebyshev Methods. Rate of convergence of iterative methods. Birge-Vita Method Bairstor's Method for real and complex roots. Graffe's Root Squaring Method.

3. *System of linear equations* :

Direct Methods-Grammer's rule, Gauss elimination, triangularization. Iterative Methods-Gauss-Jacobi Gauss-Siedel, Successive overrelaxation methods and their convergence.

Eigenvalues & eigenvectors—Garshgorin theorem, Jacobi method for symmetric matrices, Rutishaiser method for arbitrary matrices, Power method.

4. *Numerical Differentiation and Integration* :

Numerical Differentiation-Methods based on interpolation, Methods based on finite differences, Methods based on undetermined c oefficients. Optimum choicef step length. Extrapolation methods.

Numerical integration-trapetoidal Simpson's 1/3 and 3/8 rules, Weddle's rule. Gauss quadrature formula.

5. *Solutions of Ordinary Differential Equations*:

Introduction to Numerical Methods. Solutions of Difference Equations. Initial and Boundary Value Problems. Lipschitz Condition. Picards Method. Euler's Method Backward Euler's Method. Mid-Point Method. Taylor's series Method.. Runge-Kutta Method. Stability of single step methods.

Finite Difference Methods for BVP.

Suggested References :

1. Introduction to Numerical Analysis. K.E. Atkinson (John. Wiley & Sons.)
2. Introduction to Numerical Analysis C.E. Eroberg (Addison-Wesley)

3. Numerical Methods for Scientific & Engineering Computation. Jain, Iyengar & Jain (Wiley-Eastern)
4. A First Course in Numerical Analysis Ralston & Rabinowitz (McGraw-Hill)
5. Numerical Mathematical Analysis J. Scarborough
6. Introduction to Numerical Analysis Hilderbrand

Option (iv)—Linear Programming and Theory of Games:

Linear Programming: Convex sets and their properities. Theory of simplex method, Revised simplex algorithm. Degeneracy Duality theory, Sensitivity Analysis, Parametric linear programming. Transportation and Assignment problems.

Theory of Games: Rectangular Games, Saddle Points, Mixed strategies, Fundamental Theorem for rectagular games, Properties of optimal strategies. Relations of dominance. Various methods for solving rectangular games. Inter-relation between the theory of games and linear programming.

Books for References :

1. G. Hadley : Linear Programming
2. S. I. Gass ; Linear Programming: Methods Applications.
3. McKinsey : Introduction to the Theory of Games.

Option (v)—Lattice Theory :

Partial order, Chains, Lattices, Examples of Lattices. Meets and Joins, Duality, Length and Covering conditions. Atoms, complements. Complemented and relatively complemented Lattices, Sublattices, Modular and semi-modular lattices. Lattices of groups and modules.

Distributive lattices. Irreducible elements. Ideal of a lattice, Homomorphism. Isomorphism. Dual Isomorphism. Boolean Algebras.

Books for Reference :

Thomas Dounellan : Lattice Theory, Pergamon Press, Oxford, 1968.

(Chapter 2, 3, 4,; Chapter 2, sections 21, 22 and 27 only)

Option (vi)—Probability Theory :

Probability spaces. Finite Probability space. Conditional Probability, Bay's theorem, Random variables Mathematical Expectation and Moments. joint Distributions Independent Random variables. Convergence of sequence of random variables, convergence. in distributions, convergence in probability, almost sure convergence, convergence in quadratic mean, Helley-Bary Theorem. Complex valued Random variables. Characteristic function, Inversion theorem. Continuity theorem. Distribution of X and S. Kolmogorov's inequality. Weak and strong law of Large Numbers.

Books for Reference :

1. Modern Probability Theory and its applications : E. Parzon.
2. An Introduction to Probability Theory and its applications Vol. I-(3rd edition). W. Feller.
3. Probability. Elements of the Mathematics Theory : E. R. Heathcote.

Option (vii)—Computer Mathematics :

1. *Mathematical Logic* :

Statements; connectives; arguments; quantifiers; deductive and inductive methods; predicates, open and close statements.

2. *Combinational Logic* :

Basic switching functions; composite functions; analysis of logical circuits. Postulates of Boolean Algebra; De Morgans' Theorem; Duality Principle; algebraic simplifications; minimization by Karnaugh's maps and Quine-McCloskey technique.

3. *Data Representation/Binary Arithmetic* :

Conversion from decimal to binary, octal, hexadecimal numbers and vice versa. Fixed point representation, addition, substraction by 1's and 2's complement, multiplication and division; Floating point representation: BCD, ASSII, and EBCDIC codes.

4. *Optimization* :

Principle of optimization and its application to multistage problems.

5. *Problem solving on Computer* :

Flow charts and algorathms; design of programs; characterstics of a good program; top-down, bestom-up and structured programming methodologies.

6. *Programming Languages* :

Basic and PASCAL. Constants, variables, expressions, asisgnment statements, I/o constructions, Control and iterative statements arrays, sub-programs and their use in developing large programs.

7. *Development of programs for elementary problems and their implementation.*

Notes :

1. Sec. 1-4 constitute half of the paper as basic computer science and Sec. 5-7 constitute the other half as programming skill and its applications.
2. Theory paper will consist of 80 marks and 20 marks are proposed for internal assesment based upon the practical work done.
3. For theory 4 periods and for practical work 2 periods are to be allotted per week.

References :

1. Computer Hardware and Organization. M. E. Sloan (Galgotia)
2. Computer System Architecture. M. M. Mano (PHI)
3. Modern Applied Algebra Birkhoff & Bartee (CBS)

4.	Finite Mathematics	Schaum Series
5.	An Introduction to Digital Computer Design	V. Rajaraman & T. Radhakrishnan (PHI)
6.	Flow Charting, Programming software Design & Computer Problem Solving	Gayer (Wiley)
7.	Programming with PASCAL	Gottfried (McGraw Hill)
8.	Computer Programming in PASCAL	Rajaraman (PHI)
9.	PASCAL: Vseas Manual & Report	N. Wirth (Narosa Pub.)
10.	Programming with BASIC	Gottfried (McGraw Hill Book Co.)

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